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Final Project: Written Proof

Let A, B, C, and D be four sets. Prove the following two statements.

* 1. Let A, B, C, and D be sets. Suppose that A ∩ B ⊆ C \ D. Prove that if x ∈ A and x ∈ D, then x ∉ B.

Suppose that A B C D. If A and D, then B.

Suppose that A B C D. We will prove that if A and D, then B.

Let’s take the contrapositive of the statement: if B, then A or D.

Let’s assume that B. We can restate A or D as the logically equivalent statement of if A then D. So let’s assume that A.

Since we assume is in both A and B then this means that AB.

Since A B and since A ∩ B ⊆ C \ D, this means that C \ D.

Since C \ D then it follows that D.

This shows that if B then A or D. This means that the contrapositive is true that A and D then B.

* 1. A × (B ∆ C) = (A × B) ∆ (A × C).

A (B ∆ C) = (A B) ∆ (A C)

Let A, B, C, D, E, F, and G be sets.

First, we will show that a cartesian product can be distributed over a set difference.

Let be an arbitrary element of E (F G).

Then by the definition of Cartesian product, = () for some E and F G.

Since F G, F and G.

Since E and F, then it follows that E F, and since E and G, then it follows that E G.

Thus, (E F) (E G). Since was an arbitrary element of E (F G), it follows that E (F G) (E F) (E G).

Now let be an arbitrary element of (E F) (E G).

Then E F, so for some E and F.

Also, E G. Since we know already that E, then for to not be in E G, must not be in G, so G.

Since F and G, F G.

Thus, E (F G).

Since was an arbitrary element of (E F) (E G) we can conclude that (E F) (E G) E (F G), so E (F G) (E F) (E G).

Thus, we have shown that a Cartesian Product can be distributed over a set difference.

Now let’s take the left-hand side of the equation in our theorem:

A (B C).

By the definition of a symmetric difference, we know;

A (B C) A ((B C)(B C))

By the property of Cartesian Product Distributed over Set Difference, which we just showed earlier in the proof we know;

A (B C)(B C) (A (B C))(A (B C))

By the property of Cartesian Product Distributed over Union we know;

(A (B C))(A (B C))((A B)(A C))(A (B C))

By the property of Cartesian Product Distributed over Intersection we know;

((A B)(A C))(A (B C)) ((A B)(A C))((A B)(A C))

Now let’s take the right-hand side of the equation from theorem:

(A B) ∆ (A C)

By the definition of symmetric difference we know;

(A B) ∆ (A C)((A B)(A C))((A B)(A C))

We have shown that the both the left side and the right side are equal to

(A B)(A C)(A B)(A C). Which proves that;

A (B ∆ C) = (A B) ∆ (A C).